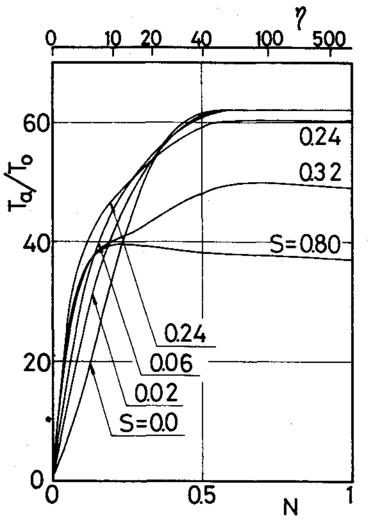

Fig. 1 Atom temperature profiles.

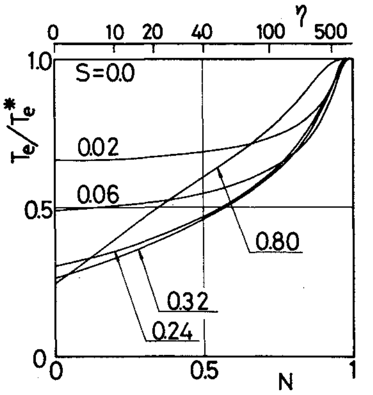

Fig. 2 Electron temperature profiles.

electron energy are ignored in the boundary layer. The two-layer structure of the flow is clearly seen. The thickness of the electron thermal layer is found to be $\eta = 500$ or above, while the thickness of the viscous boundary layer varies between $\eta = 10$ and $\eta = 40$. Thus, it is assured that the detailed structure of both the thin viscous boundary layer and the thick electron thermal layer can easily be obtained by using the reduced coordinates (S, N).

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Integral Momentum Equation for Flows with Entropy Gradients across Inviscid Streamlines

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Nomenclature

h	= enthalpy
H	= total enthalpy, $h+u^2/2$
p	= pressure
r	= normal distance from the body axis to the surface
s	= entropy
T	= temperature
u	= streamwise velocity
v	= velocity normal to the surface
x	= streamwise coordinate along the surface
y	= coordinate normal to the surface
δ^*	= displacement thickness, defined by Eq. (18)
δ^{**}	= displacement of inviscid streamlines by the boundary layer
ϵ	= geometry index for axisymmetric or two-dimensional flow
θ	= momentum thickness, defined by Eq. (17)
μ	= dynamic viscosity
ρ	= mass density
τ	= shear stress
ψ	= stream function, defined by Eq. (8)
Superscripts	
$'$	= fluctuating quantities
$-$	= mean quantities
$+$	= specific streamline outside of boundary layer
Subscripts	
e	= normal shock edge conditions
i	= inviscid
w	= surface
∞	= outside of boundary layer

Introduction

In the flowfield around a blunt axisymmetric or two-dimensional body, the entropy of the fluid following a streamline of the inviscid flow is determined by upstream conditions and the local angle of the shock through which the streamline passes. Curved shocks therefore generate entropy gradients in the inviscid flow which, in turn, result in inviscid velocity gradients normal to the surface. The boundary-layer integral momentum equations and appropriate definitions of the momentum and displacement thicknesses valid for this situation are developed in this paper.

Inviscid Flowfield

The total enthalpy H is assumed to be constant throughout the inviscid flowfield, so that an energy balance yields

$$h_i + (u_i^2/2) = H_i \tag{1}$$

and

$$dh_i = -u_i \, du_i$$

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Shock shape and surface pressure distributions are assumed to be known. With pressure and entropy considered to be independent thermodynamic variables, differential changes in enthalpy satisfy the relationship

$$dh=(1/\rho)dp+T\,ds \tag{2}$$

By combining Eqs. (1) and (2), the acceleration of a fluid element along an inviscid streamline $[(\partial s_i/\partial x)_\psi=0]$ is found to be

$$\rho_i u_i(\partial u_i/\partial x)_\psi=-dp/dx \tag{3}$$

For the portion of the flow which becomes the viscous boundary layer, it has been assumed that normal pressure gradients can be neglected $[(\partial p/\partial\psi)_x=0]$.

Integral Momentum Equation

With the usual boundary-layer approximations, the equations for conservation of mass and momentum in axisymmetric ($\epsilon=1$) and two-dimensional ($\epsilon=0$) compressible turbulent boundary layers[1] are

$$\frac{1}{r^\epsilon}\frac{\partial}{\partial x}(\bar\rho\bar u r^\epsilon)+\frac{\partial}{\partial y}(\overline{\rho v})=0 \tag{4}$$

$$\frac{\bar\rho\bar u^2}{r^\epsilon}\frac{dr^\epsilon}{dx}+\frac{\partial}{\partial x}(\bar\rho\bar u^2)+\frac{\partial}{\partial y}(\bar\rho\bar u\bar v)=-\frac{dp}{dx}+\frac{\partial\bar\tau}{\partial y} \tag{5}$$

The overbar denotes mean values. Fluctuating quantities, denoted by primes, appear in two terms

$$\overline{\rho v}=\bar\rho\bar v+\overline{\rho' v'} \tag{6}$$

and

$$\bar\tau=\mu\frac{\partial\bar u}{\partial y}-\bar\rho\overline{u'v'}-\bar u\overline{\rho' v'} \tag{7}$$

When the inviscid flow is of uniform entropy, the integral momentum equation is obtained by multiplying Eq. (4) by the edge velocity, subtracting this result from Eq. (5), and integrating across the boundary layer. The first step in this process associates with each element of fluid the momentum that it would have had if there were no viscous boundary layer. The generalization of this operation is the key to the present development.

A stream function for the viscous flow may be defined as

$$\psi(x,y)=-\int_0^x \rho_w v_w r^\epsilon dx+r^\epsilon\int_0^y \bar\rho\bar u\,dy \tag{8}$$

and

$$\left[\frac{\partial\psi}{\partial y}\right]_x=\bar\rho\bar u r^\epsilon,\quad\left[\frac{\partial\psi}{\partial x}\right]_y=-\overline{\rho v}\,r^\epsilon \tag{9}$$

The stream function identifies the streamline passing through any particular point in the viscous boundary layer and determines the velocity that the fluid element at that point would have had in the absence of viscosity, $u_i[x,\psi(x,y)]$. This identification is correct only in an average sense for turbulent flow.

The traditional process for obtaining the integral momentum equation may now be followed with $u_i\ [x,\psi(x,y)]$ used in place of the edge velocity. When $\rho_w v_w>0$, the inviscid velocity field is extended to $\psi<0$ by assigning a constant velocity $u_i(x,\psi=0)$ to this region. This step is implicit in the traditional development. The result of multiplying Eq. (4) by $u_i[x,\psi(x,y)]$ is

$$\frac{\partial}{\partial x}(\overline{\rho v}\,u_i)+r^\epsilon\frac{\partial}{\partial y}(\overline{\rho v}\,u_i)=\bar\rho\bar u r^\epsilon\frac{\partial u_i}{\partial x}+\overline{\rho v}\,r^\epsilon\frac{\partial u_i}{\partial y} \tag{10}$$

The terms on the right-hand side of this equation may be identified as a derivative along a streamline. Considering, for the time being, (x,ψ) as independent variables rather than (x,y), the chain rule gives [using Eqs. (9)]

$$\left[\frac{\partial}{\partial x}\right]_y=\left[\frac{\partial}{\partial x}\right]_\psi-\overline{\rho v}\,r^\epsilon\left[\frac{\partial}{\partial\psi}\right]_x,\ \left[\frac{\partial}{\partial y}\right]_x=\bar\rho\,\bar u\,r^\epsilon\left[\frac{\partial}{\partial\psi}\right]_x \tag{11}$$

from which it is apparent that

$$\bar\rho\bar u r^\epsilon\left[\frac{\partial u_i}{\partial x}\right]_y+\overline{\rho v}\,r^\epsilon\left[\frac{\partial u_i}{\partial y}\right]_x=\bar\rho\bar u r^\epsilon\left[\frac{\partial u_i}{\partial x}\right]_\psi \tag{12}$$

Equations (3), (10), and (12) are combined and Eq. (5) is subtracted from the resulting equation. This gives

$$\frac{1}{r^\epsilon}\frac{dr^\epsilon}{dx}\bar\rho\bar u(\bar u_i-\bar u)+\frac{\partial}{\partial x}[\bar\rho\bar u(u_i-\bar u)]+\frac{\partial}{\partial y}[\overline{\rho v}(u_i-\bar u)+\overline{\rho' v'}\,\bar u]=-(\rho_i u_i-\bar\rho\bar u)\left[\frac{\partial u_i}{\partial x}\right]_\psi-\frac{\partial\bar\tau}{\partial y} \tag{13}$$

For the purposes of rendering the equation dimensionless, a characteristic velocity to be used in the role of the edge velocity is required. Only one choice is available; the inviscid velocity associated with the normal shock entropy. Accordingly, an "edge velocity" is defined as

$$u_e(x)=u_i(x,\psi=0) \tag{14}$$

and similarly

$$\rho_e(x)=\rho_i(x,\psi=0) \tag{15}$$

Now, since the "edge" inviscid streamline is just a special case for which Eq. (3) holds, the velocity gradient along any inviscid streamline may be written as

$$\left[\frac{\partial u_i}{\partial x}\right]_\psi=\frac{\rho_e u_e}{\rho_i u_i}\frac{du_e}{dx} \tag{16}$$

This result is substituted for the inviscid velocity gradient in Eq. (13), which is then in the desired form to be integrated across the boundary layer.

Anticipating the results of integrating Eq. (13), the definitions of the momentum and displacement thicknesses are generalized to

$$\theta=\int_0^\infty\frac{\bar\rho\bar u}{\rho_e u_e}\left[\frac{u_i-\bar u}{u_e}\right]dy \tag{17}$$

and

$$\delta^*=\int_0^\infty\left[1-\frac{\bar\rho\bar u}{\rho_i u_i}\right]dy \tag{18}$$

With these definitions, integration of Eq. (13) across the boundary layer yields the standard dimensionless form of the integral momentum equation, which is

$$\frac{\theta}{r^\epsilon}\frac{dr^\epsilon}{dx}+\frac{1}{\rho_e u_e}\frac{d}{dx}(\rho_e u_e\theta)+\frac{(\theta+\delta^*)}{u_e}\frac{du_e}{dx}=\frac{\rho_w v_w}{\rho_e u_e}+\frac{\tau_w}{\rho_e u_e^2} \tag{19}$$

In keeping with the approximation involved in the calculation of the inviscid flowfield, the term $\bar{\tau}_\infty$ has been neglected.

Apparent Body Surface Displacement

The displacement of the inviscid dividing streamline ($\psi = 0$) which will align streamlines in the inviscid flow with corresponding streamlines outside the boundary layer in the actual flow is designated here to be $\delta^{**}(x)$. This displacement can be related to the displacement thickness [$\delta^*(x)$, defined by Eq. (18)] as is shown in the following.

Consider some particular streamline ψ^+ outside the boundary layer. Its location may be computed from

$$y^+ = \frac{1}{r^\epsilon}\int_{\psi_w}^{\psi^+}\frac{d\psi}{\bar{\rho}\bar{u}} \tag{20}$$

Similarly, for the inviscid flow

$$y^+ - \delta^{**} = \frac{1}{r^\epsilon}\int_0^{\psi^+}\frac{d\psi}{\rho_i u_i} \tag{21}$$

Also, since for $\psi < 0$, $\rho_i u_i = \rho_e u_e$

$$0 = \frac{1}{r^\epsilon}\int_{\psi_w}^{0}\frac{d\psi}{\rho_i u_i} + \frac{\psi_w}{\rho_e u_e r^\epsilon} \tag{22}$$

Equations (21) and (22) are subtracted from Eq. (20) giving

$$\delta^{**} = \frac{1}{r^\epsilon}\int_{\psi_w}^{\psi^+}\left[\frac{1}{\bar{\rho}\bar{u}} - \frac{1}{\rho_i u_i}\right]d\psi - \frac{\psi_w}{\rho_e u_e r^\epsilon} \tag{23}$$

The integral may now be transformed using $d\psi = \bar{\rho}\bar{u}\, r^\epsilon dy$, so that

$$\delta^{**} = \delta^* - (\psi_w/\rho_e u_e r^\epsilon) \tag{24}$$

This is the usual result except for the more general definition of δ^*.

Conclusions

The integral momentum equation has been generalized to include flows with entropy gradients across inviscid streamlines. A related equation for total enthalpy of the mean flow ($H = h + u^2/2$) can also be obtained. However, since its development is unaffected by such entropy gradients, it is not discussed herein. The generalized integral momentum equation together with the integral total enthalpy equation should be useful in calculating approximate boundary-layer solutions in flows which have passed through curved shocks.

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Noise Produced by Fluid Inhomogeneities

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I. Introduction

THERE is currently in progress a substantial amount of research concerning the noise production of hot spots passing through extreme velocity gradients. This process is found in turbopropulsion systems where hot spots produced in the combustor pass through nozzles in the turbine assembly. The mechanism of sound production by this process has been known for some time, was explicity considered in numerous investigations of entropy wave instability in rocket engines and motors, and was quantitatively presented in the calculations of Crocco and Sirignano.[1] The mechanism was considered by Cuadra,[2] and, more recently, was investigated concerning aeroacoustics implications by Candel.[3] Research along these lines has continued through the works of Zukoski[4] and Cumpsty,[5] and an application of the theory to core engine noise from an engine has been made by Pickett.[6] This kind of noise has been called entropy noise by the author[7] to distinguish it from noise directly generated by the turbulent combustion process, which may form another significant component of core engine noise.

Entropy noise from hot spots is a degenerate form of noise that may be caused by various kinds of fluid inhomogeneities. There are at least two basic mechanisms that will cause noise when a fluid element of different thermodynamic properties from the surrounding fluid attempts to traverse a given pressure (or velocity) gradient. The first is that the acceleration of the element in question may be different from the surrounding fluid by virtue of a different density. This is seen easily by examination of the momentum equation $Du/Dt = -\nabla p/\rho$, where D/Dt is the material time derivative $\boldsymbol{u}$ is the velocity vector and $\nabla p/\rho$ is the pressure gradient term. Clearly, ρ, the density, may be affected by temperature or molecular weight at a given value of the pressure. Inhomogeneities in the temperature and the molecular weight will give rise to different accelerations of neighboring elements, and an unsteady motion must be set up to alter ∇p. This alteration, of course, causes sound to be generated. A second mechanism of noise generation, and one considered explicitly in this paper, has not appeared to have been explored, however; this is caused by inhomogeneities in specific heat. For isentropic motion of perfect gases $(D\rho/Dt)(1/\rho) = (Dp/Dt)/(\gamma p)$. Consequently, for a fixed pressure change, the fractional change in density depends upon γ, the ratio of specific heats. Inhomogeneities in this quantity would give rise to separation of fluid elements, which cannot be allowed by continuity considerations. Again, an unsteady adjustment must take place which is perceived as sound.

In an actual engine system, one mechanism for production of hot spots is the burning of various fluid elements at various different mixture ratios. Since a gas turbine combustor burns by a diffusion flame, whereby the air and fuel must first mix before they burn, there is no guarantee that an absolutely uniform air/fuel ratio can be maintained for every fluid element. Variable mixture ratio means, of course, variable temperature, and this is the effect that has been studied insofar as a noise source is concerned. However, variable mixture ratio also implies variable molecular weight and heat capacity. The purpose of this Note is to investigate whether either of these last two variations may be responsible for a significant noise source.

II. Analysis

The analysis will be done within the context of one-dimensional unsteady flow, as in the work of Candel.[3] A mixture of thermally perfect gases will be assumed as the working fluid. The fluid composition will be considered to consist of species 1 (which may also be a mixture) and a small and variable mole fraction of species 2, which has different molecular weight and specific heat as compared with species 1. Axial diffusion of species is assumed to be negligible, so that the space-wise variation of composition is convected by the flow velocity. Furthermore, small density variations are allowed due to temperature and molecular weight variations; there is also no axial diffusion of heat, and so the hot spots (or cold spots) are convected by the flow. One considers,

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